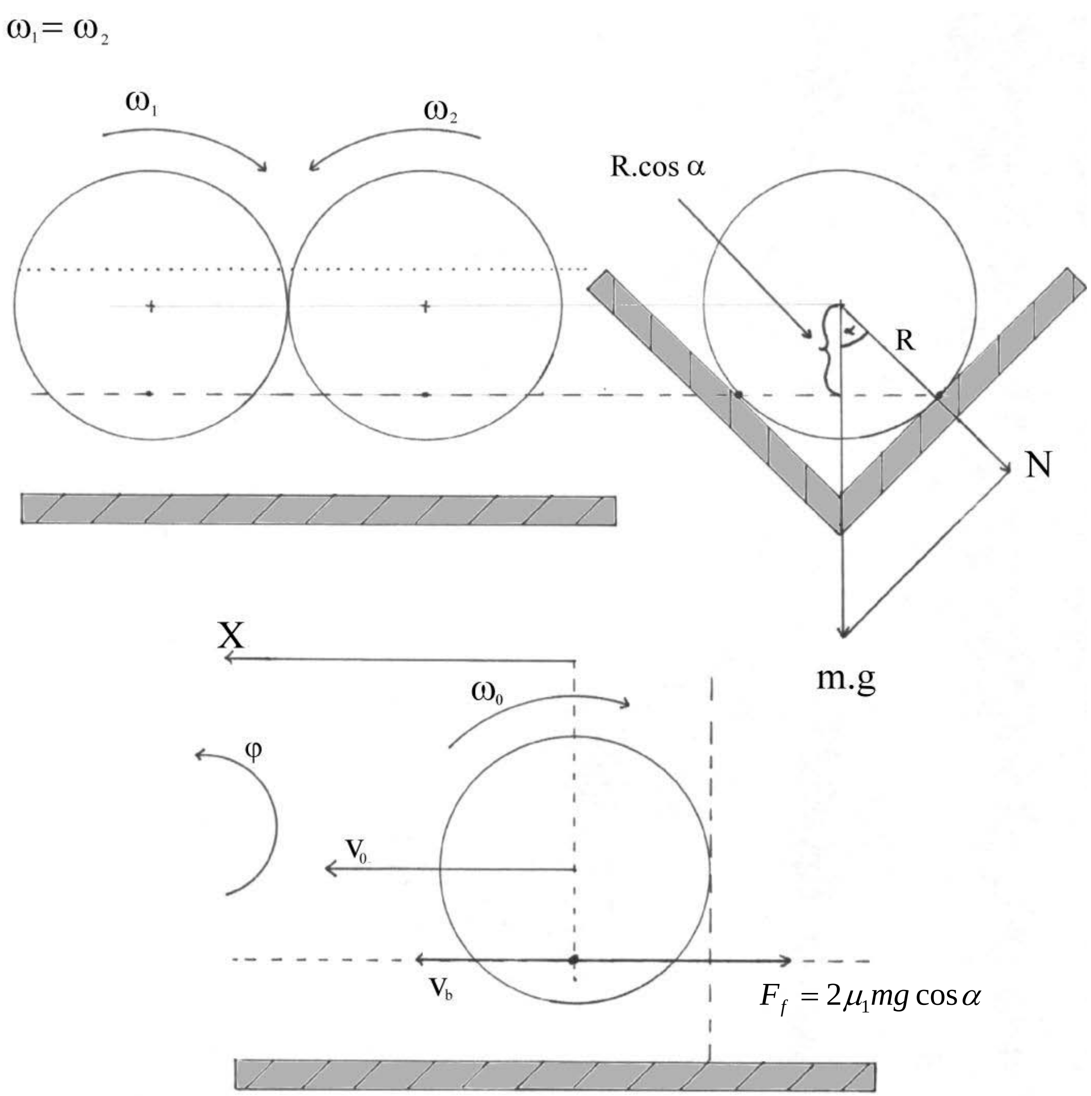
Problem no. 11 Rolling Balls

Place two equal balls in a horizontal V-shaped channel, with the walls at 90° to each other, and let the balls roll towards each other. Investigate and explain the motion of the balls after the collision. Make experiments with several different kinds of ball pairs and explain the results.

Case 1: $\omega_1 = \omega_2$ ($\Rightarrow$ no friction during collision)



Equation 1

$$m \cdot \ddot{x} = -2 \cdot F_f = -2 \cdot \mu_1 \cdot N$$

μ_1...coefficient of friction between ball and channel

$$m \cdot \ddot{x} = -2 \cdot \mu_1 \cdot m \cdot g \cdot \cos\alpha = -2 \cdot \mu_1 \cdot m \cdot g \cdot \cos 45°$$

$$\ddot{x} = -\sqrt{2} \cdot \mu_1 \cdot g$$

$$\dot{x} = -\sqrt{2} \cdot \mu_1 \cdot g \cdot t + const.$$

$$t = 0: \quad \dot{x} = v_0$$

$$\boxed{\dot{x}(t) = v_0 - \sqrt{2} \cdot \mu_1 \cdot g \cdot t}$$

Equation 2

$$I_s \cdot \ddot{\varphi} = M = 2 \cdot F_f \cdot R \cdot \cos\alpha$$

μ_1...coefficient of friction between ball and channel

$$\frac{2}{5} \cdot m \cdot R^2 \cdot \ddot{\varphi} = 2 \cdot \mu_1 \cdot m \cdot g \cdot \cos^2\alpha \cdot R$$

$$\ddot{\varphi} = \frac{5 \cdot \mu_1 \cdot g}{2 \cdot R}$$

$$\dot{\varphi} = \frac{5 \cdot \mu_1 \cdot g}{2 \cdot R} \cdot t + const.$$

$$t = 0: \quad \dot{\varphi} = -\omega_0$$

$$\boxed{\dot{\varphi} = \frac{5 \cdot \mu_1 \cdot g}{2 \cdot R} \cdot t - \omega_0}$$

Condition for pure rolling towards each other

$$-\dot{x} = R \cdot \cos\alpha \cdot \dot{\varphi}$$

$$\boxed{\tau_1 = \frac{\dfrac{R \cdot \omega_0}{\sqrt{2}} - v_0}{0{,}354 \cdot \mu_1 \cdot g}}$$

τ_1 is the time, after which the balls start rolling towards each other again after the collision.

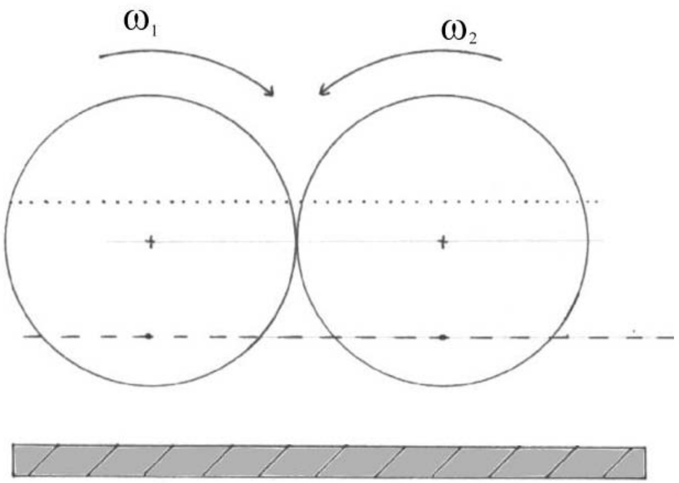
Condition for pure rolling away from each other

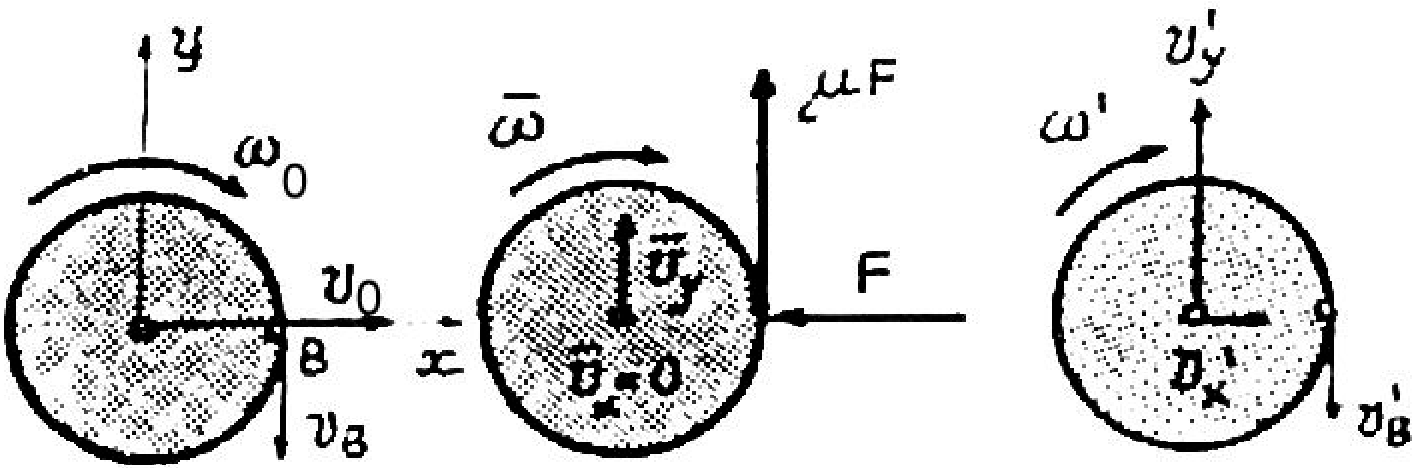
$$\dot{x} = R \cdot \cos\alpha \cdot \dot{\varphi}$$

$$\tau_2 = \frac{\dfrac{R \cdot \omega_0}{\sqrt{2}} + v_0}{3{,}18 \cdot \mu_1 \cdot g}$$

τ_2 is the time, after which the balls start rolling purely away from each other after the collision.

Case 2: $\omega_1 > \omega_2$ ($\Rightarrow$ friction between the balls)





Course of the impact force

$$m \cdot (0 - v_0) = -\int_0^{\bar{t}} F \cdot dt = -S_1$$

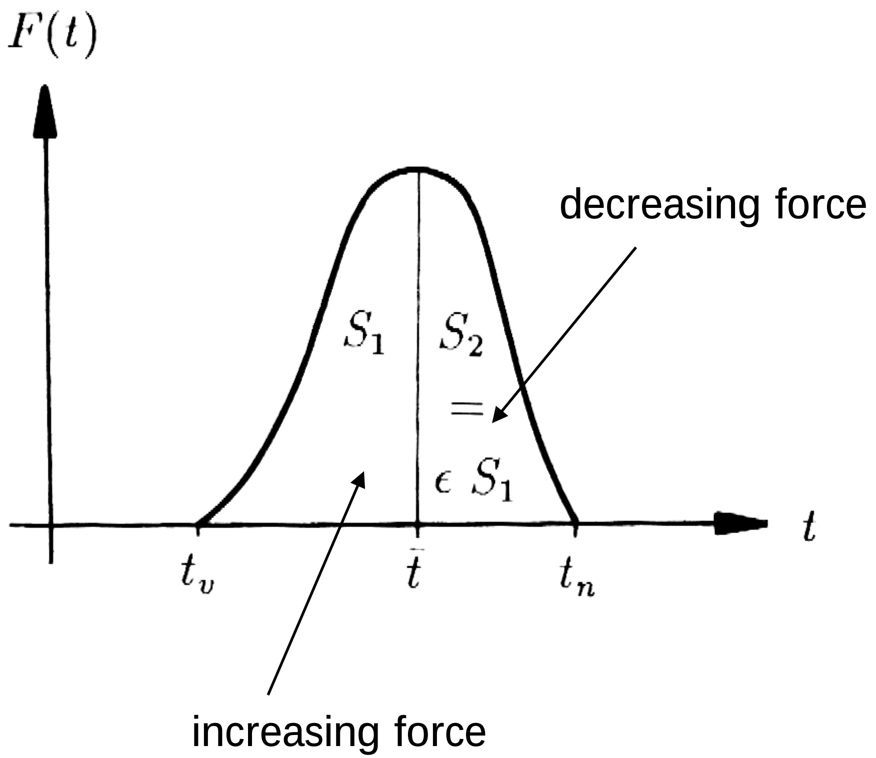
$$m \cdot (v_x' - 0) = \int_{\bar{t}}^{t_E} F \cdot dt = -S_2$$

$$m \cdot \left(\bar{v}_y - 0\right) = \int_0^{\bar{t}} \mu_2 \cdot F \cdot dt = \mu_2 \cdot S_1$$

$$m \cdot \left(v_y' - \bar{v}_y\right) = \mu_2 \cdot S_2$$

$$I_S \cdot (\bar{\omega} - \omega_0) = -\mu_2 \cdot \int F \cdot R \cdot dt = -\mu_2 \cdot S_1 \cdot R$$

$$I_S \cdot (\omega' - \bar{\omega}) = -\mu_2 \cdot S_2 \cdot R$$



μ_2...coefficient of friction between the two balls

Completely elastic collision

$$\varepsilon = \frac{S_2}{S_1} = 1$$

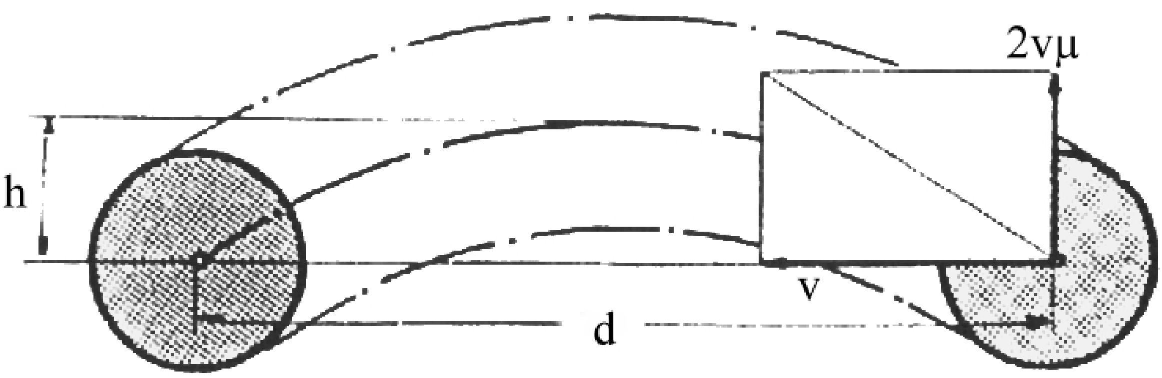
$$v_x' = -v_0$$

$$v_y' = 2 \cdot \mu_2 \cdot v_0$$

$$\omega' = \frac{v_0}{R} \cdot (1 - 5 \cdot \mu_2)$$

$$d = \frac{4 \cdot \mu_2 \cdot v_0^2}{g}$$

$$h = \frac{4 \cdot \mu_2^2 \cdot v_0^2}{g}$$



Partly elastic/plastic collision

$$d = \frac{2 \cdot v_0^2 \cdot \mu_2}{g} \cdot \varepsilon \cdot (1 + \varepsilon)$$

$$h = \frac{\mu_2^2 \cdot v_0^2}{2 \cdot g} \cdot (1 + \varepsilon)^2$$

$\varepsilon = S_2/S_1$....coefficient of collision

Experimental setup