# IYPT 2000
# Problem No. 3
# PLASMA

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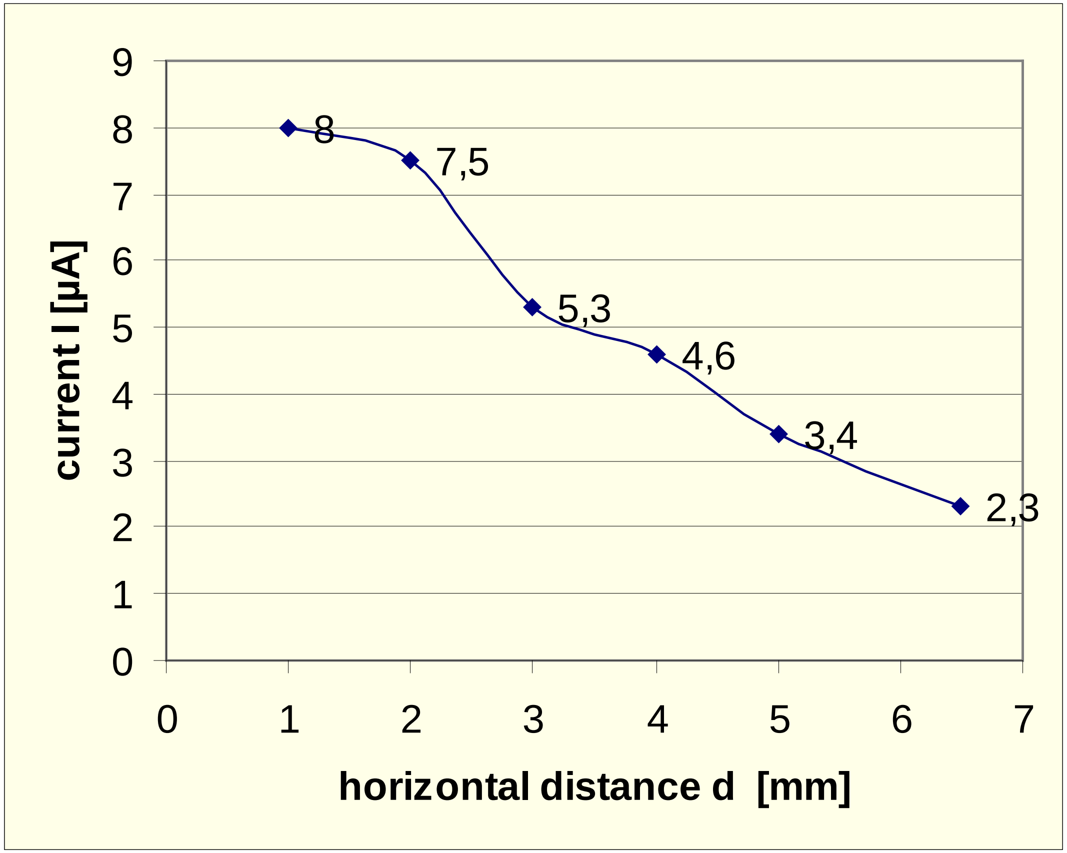
Team Austria

Investigate the electrical conducivity of the flame of a candle. Examine the influence of relevant parameters, in particular, the shape and polarity of the electrodes. The experiments should be carried out with a voltage not exceeding 150 V.

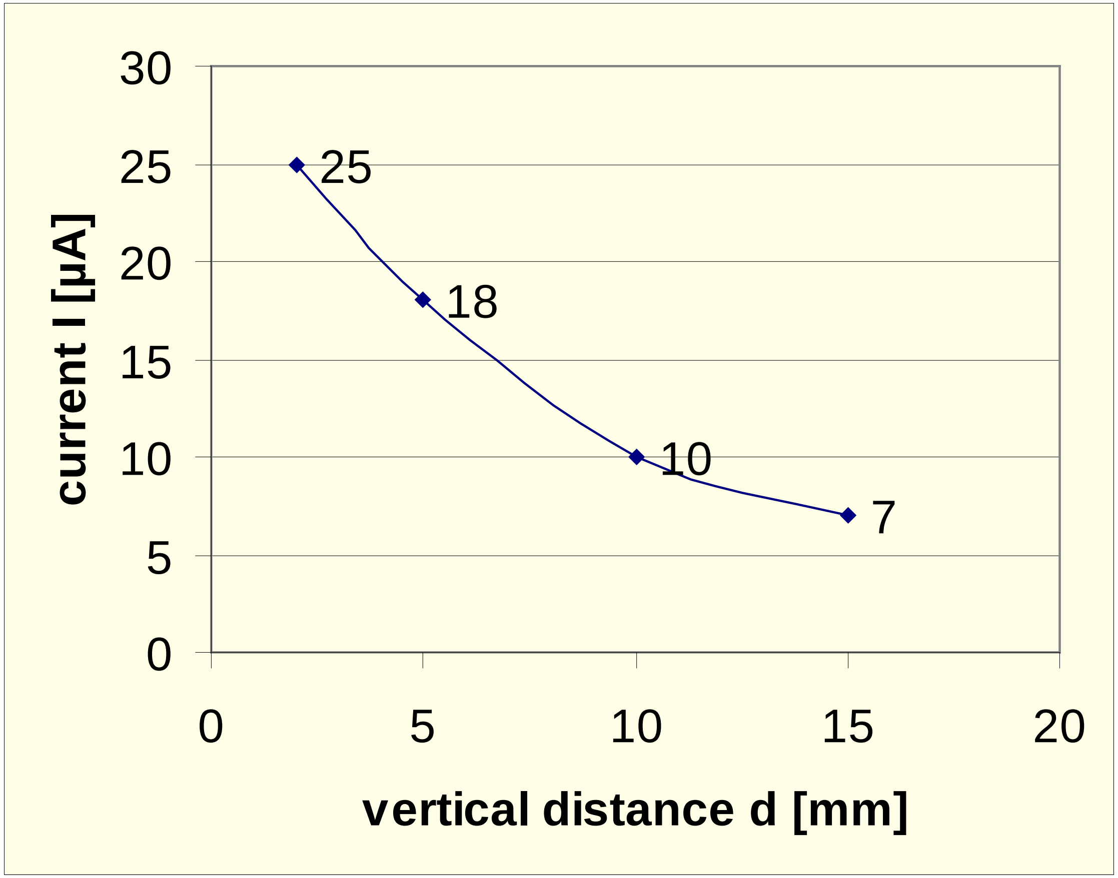
# Measurements



***experimental data:***

- sharp electrodes
- on the same level in the flame
- voltage: 150 V, DC

# Measurements



***experimental data:***

- sharp electrodes
- above each other
- Intersection: about 4 mm
- voltage: 150 V, DC

# Conclusions from the measurements

The problem is ***not*** completely **solveable**!

Only the influence of various paramters can be examined:

- electric field (shape and arrangement of the electrodes)
- temperature
- gas flow
- compositon of the gas

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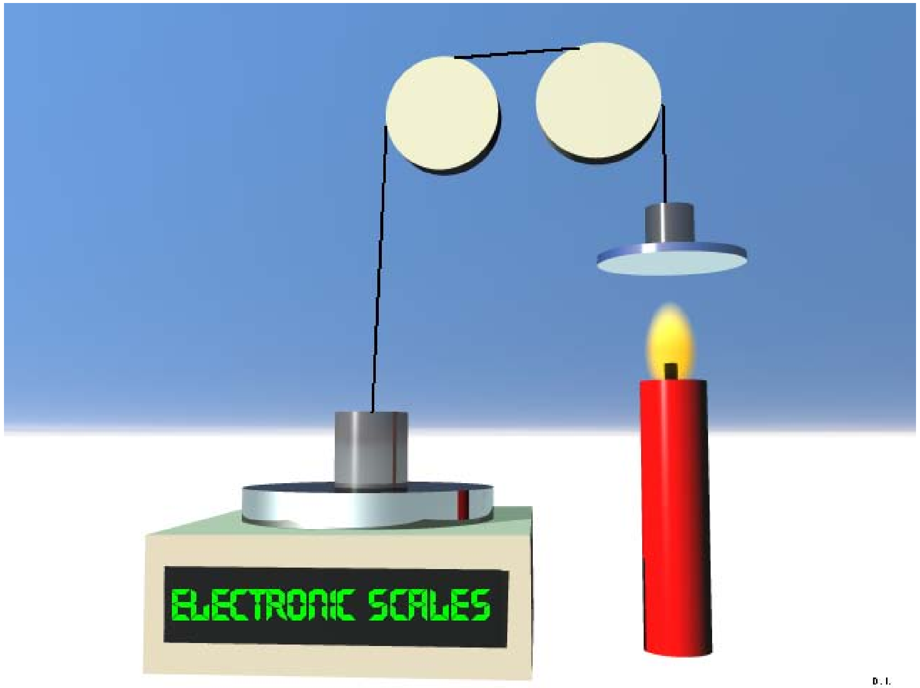
# Gas Flow - Experiment

left mass: 2g; right mass: 1g; connected with a circular **Al** – plate (in the flame: **Fe** – plate);Temperature measurement with Pt-100 or thermocouple

The streaming force $\mathbf{F}_{\mathbf{Str}}$ is an upward force on the right mass and produces an additional downward force on the electronic scales. This causes the scales to show a higher weight, so $\mathbf{F}_{\mathbf{Str}}$ can be calculated using this “mass difference” $\mathbf{\Delta m}$.



# Gas Flow - Calculations

$$F_{Str} = \frac{c_w \cdot A \cdot \rho \cdot v^2}{2} \Rightarrow v = \sqrt{\frac{2 \cdot F_{Str}}{c_w \cdot A \cdot \rho}}$$

$C_w$ = 1,1 (for circular plates)
A = r².π (area of the circular plate)
$F_{Str}$ ... has been measured
ρ ... density of air: calculated (see following transparent)

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***example of calculation:*** distance from the flame: 50 cm

$$\Delta m = 5 \cdot 10^{-6} [\mathrm{kg}]$$

$$F_{Str} = 4{,}905 \cdot 10^{-5} [\mathrm{N}]$$

$$\vartheta = 22{,}5°\mathrm{C} \Rightarrow \rho = 1{,}195 \left[\frac{\mathrm{kg}}{\mathrm{m}^3}\right]$$

$$v = \sqrt{\frac{2 \cdot F_{Str}}{c_w \cdot A \cdot \rho}} = \sqrt{\frac{2 \cdot 4{,}905 \cdot 10^{-5}}{1{,}1 \cdot 1{,}256 \cdot 10^{-3} \cdot 1{,}195}}$$

$$v \approx 0{,}24 \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

# Gas Flow - Calculations

## **Calculation of air density:** ideal gas equation

$$p \cdot V = n \cdot R \cdot T$$

$$p \cdot V = \frac{m}{M} \cdot R \cdot T$$

$$p \cdot M = \frac{m}{V} \cdot R \cdot T$$

$$p \cdot M = \rho \cdot R \cdot T$$

$$\rho = \frac{p \cdot M}{R \cdot T}$$

| | |
|---|---|
| p | pressure [Pa] |
| V | volume [m³] |
| n | number of moles |
| R | gas constant (= 8,31) [ J / (K . mol) ] |
| T | temperature [K] |
| M | molecular weight |
| ρ | density [kg/m³] |

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***calculation of the molecular weight M:***

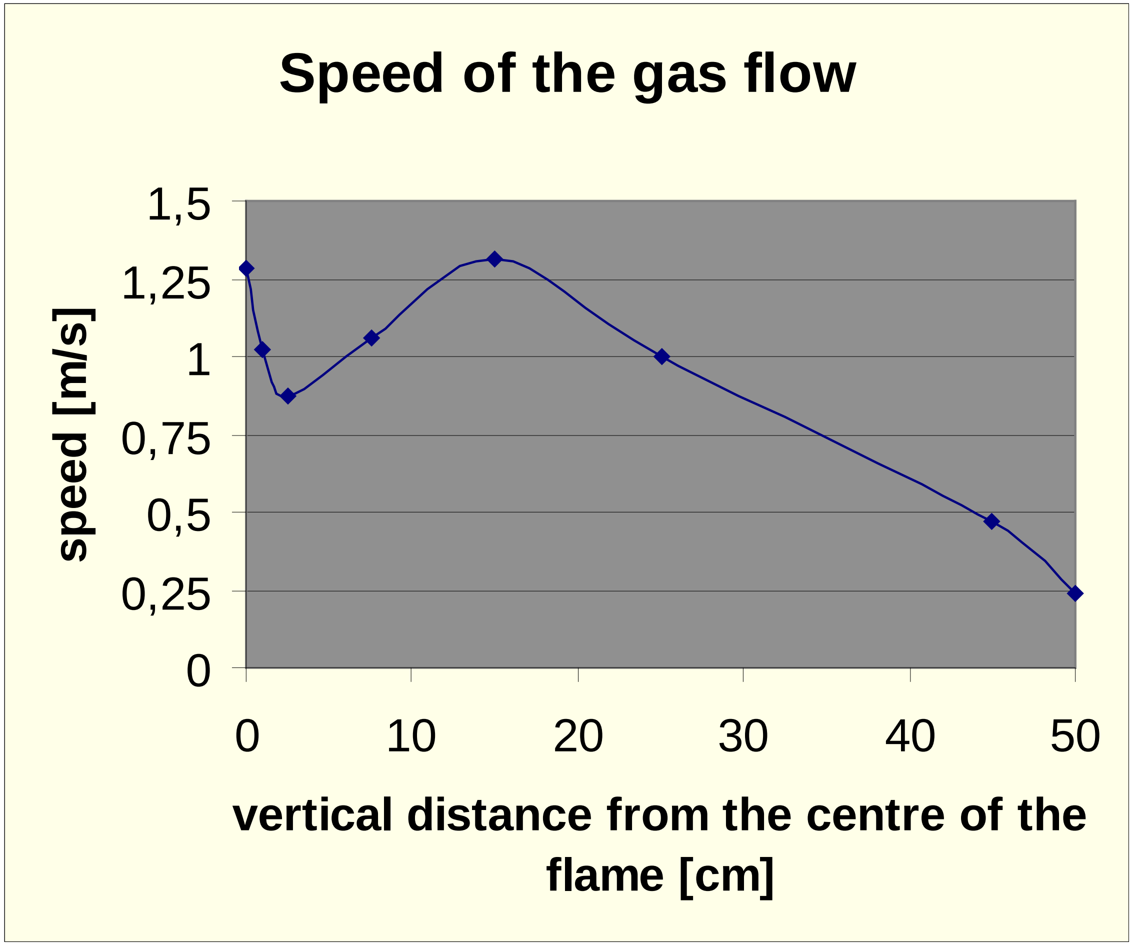
air at 0°C, 1 bar: ρ = 1,293 [kg/m³]

$$M = \frac{\rho \cdot R \cdot T_0}{p_0} = \frac{1{,}293 \cdot 8{,}31 \cdot 273{,}15}{10^5}$$

$$M = 0{,}02935[\text{kg}]$$

(ρ [kg/m³] → M [kg])

# Gas Flow - Results



# Theory about plasma

Plasma in thermic equlibrium: 3 distributions

- Boltzmann distribution
- Maxwell distribution
- Saha distribution

For a plasma in thermic equlibrium, all 3 distributions have to be fulfilled.

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# Maxwell distribution

$$N(v) = 4\pi\ N\left(\frac{m_M}{2\pi\ k\ T}\right)^{\frac{3}{2}} v^2 \cdot e^{-\frac{m_M v^2}{2kT}}$$

N…number of molecules

k…Boltzmann constant ($\approx$1,38.$10^{-23}$ [J/K])

$m_M$…mass of one molecule

T…temperature [K]

# Ionisation through hits

calculation of the required minium speed for ionisation:

$$E_i = \frac{m_M \cdot v_{min}^2}{2}$$

Maxwell distribution to calculate the fraction of molecules which are faster than $v_{min}$:

$$x = 4\pi \left( \frac{m_M}{2\pi\, k\, T} \right)^{\frac{3}{2}} \int_{v_{min}}^{\infty} v^2 \cdot e^{-\frac{m_M v^2}{2kT}}\, dv$$

N...number of molecules

k...Boltzmann constant (≈1,38.$10^{-23}$ [J/K])

$m_M$...mass of one molecule

T...temperature [K]

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results: e.g. for $O_2$: $x \approx 3.\ 10^{-64}$

for $N_2$: $x \approx 9.\ 10^{-91}$

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# Saha distribution

The Saha equation indicates the ionisation balance between the electrones and the ions versus neutral particles:

$$S = \frac{N_e^- \cdot N_i^+}{N} = \frac{\left(2 \cdot \pi \cdot m_e\right)^{\frac{3}{2}}}{h^3} \cdot \left(k \cdot T\right)^{\frac{5}{2}} \cdot e^{-\frac{E_i}{k \cdot T}}$$

$m_e$....mass of the electron

h......Planck constant

k.......Boltzmann constant

T.......temperature

$E_i$......ionisation energy

like chemical equilibrium:

$$X \rightarrow X^+ + e^-$$

Chemical equation for the ionisation of an element X.

$$K = \frac{\left[X^+\right] \cdot \left[e^-\right]}{\left[X\right]} = f\left(T, E_i\right)$$

# Influence of the electric field

The electric field causes electrons to drift through the plasma with a current density j:

$$j = N_e \cdot e \cdot v_d$$

moreover:

$$j = \sigma \cdot E$$

$$v_d = \frac{e}{m_e} \cdot \tau \cdot E$$

$$v_d = \mu \cdot E$$

$$\lambda_e = v_e \cdot \tau$$

$$v_e = \sqrt{\frac{8 \cdot k \cdot T}{\pi \cdot m_e}}$$

$N_e$...number of electrones

e....elementary charge

$v_d$...drifting speed

E....electric field strength

$m_e$..mass of the electron

$\tau$.....average time between 2 hits

$\sigma$....conductivity

$\mu$....mobility of the electrons

$\lambda_e$....free distance between 2 hits

$v_e$....average speed of the electrons (Maxwell distribution)

k.....Boltzmann constant

T.....temperature

# calculation

using the shown formulas:

$$j = N_e \cdot e \cdot \mu_e \cdot E = \sigma \cdot E$$

$$\Rightarrow \sigma = \frac{1}{2}\sqrt{\frac{\pi}{2}} \cdot \frac{e^2}{\sqrt{m_e \cdot k \cdot T}} \cdot N_e \cdot \lambda_e$$

$$\Rightarrow \sigma \sim \frac{1}{\sqrt{m}}$$

Taking an element with an average mass from the periodic system of the elements we can show that the share of ions of the current is < 0,4 %.

So we can restrict ourselves to the consideration of drifting electrones only.

# Richardson equation

$$j = A \cdot T^2 \cdot e^{-\frac{W_A}{k \cdot T}}$$

$$A = \frac{4\pi \cdot m \cdot k \cdot T}{h^3}$$

$W_A$...work function

k……Boltzmann constant

T……Temperature

h……Planck constant

m…..rest mass of the electrone

The Richardson equation shows only the influence of the temperature and is ***not considering the point effect***!

# Field influence

field emission: Schottky-effect (point effect)

Fowler-Nordheim equation:

$$E = \frac{F}{Q} = \frac{F}{e_c}$$

$$j \sim E^2 \cdot e^{-\frac{E_0}{E}}$$

E….electric field strength

F….force

Q….charge

$e_c$…elementary charge

j…..current density